Machine Learning I

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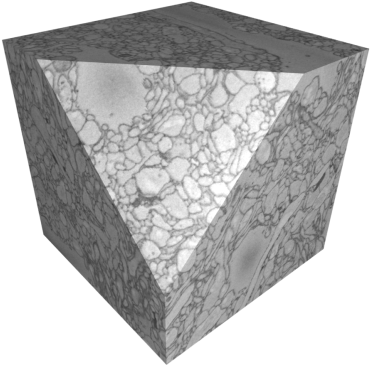
Machine Learning for Computer Vision
TU Dresden

Contents:

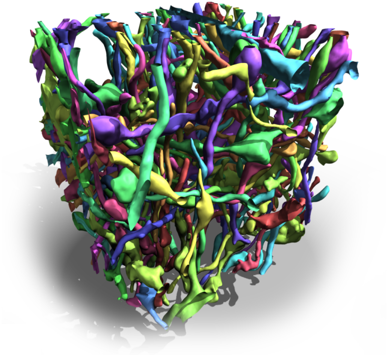
- This part of the course is about an application of clustering to the task of image decomposition.
- No additional definitions or algorithms are introduced in this lecture. Instead, this lecture illustrates the definitions and algorithms introduced in the previous lecture on clustering.

Volume Image (32 nm/voxel)
(Denk and Horstmann, 2004)

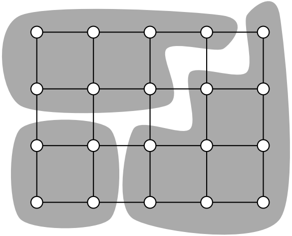


$\mapsto$

Decomposition
(Andres et al., 2012)

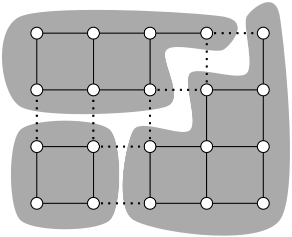


The volume image taken by a Block Face Scanning Electron Microscope shows cells that are indistinguishable by appearance. Decomposing such an image into individual cells is one challenge toward the ambitious goal of mapping the connectivity of nervous systems.



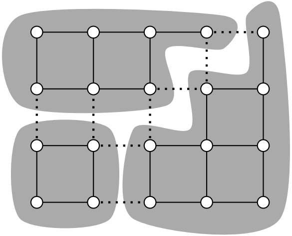
Decomposition of a graph $G = (V, E)$

- A mathematical abstraction of a decomposition of an image is a decomposition of the pixel grid graph.
- A decomposition of a graph is a partition of the node set into connected subsets (one example is depicted above in gray).



Decomposition of a graph $G = (V, E)$

- A decomposition of a graph is characterized by the set of edges that straddle distinct components (depicted above as dotted lines)
- Those subsets of edges are called **multicuts** of the graph



Multicut of a graph $G = (V, E)$

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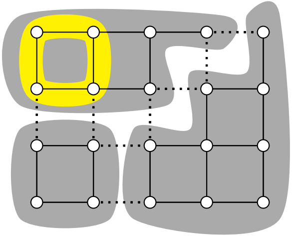
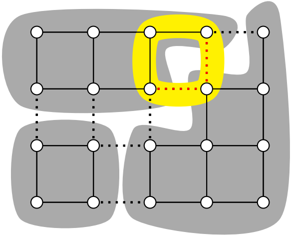


Multicut of a graph $G = (V, E)$

- The defining property of multicuts is that no cycle in the graph intersects with the multicut in precisely one edge



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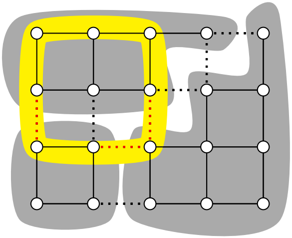

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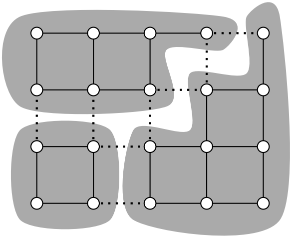
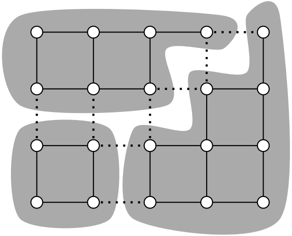


Multicut of a graph $G = (V, E)$

$$\text{multicuts}(G) := \{M \subseteq E \,|\, \forall C \in \text{cycles}(G) : \, |M \cap C| \neq 1\}$$



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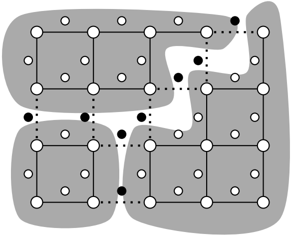
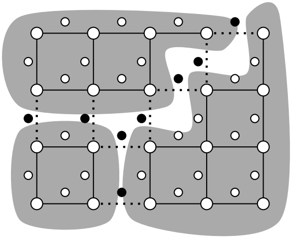


Multicut of a graph $G = (V, E)$

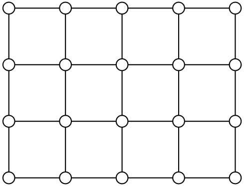
- The characteristic function $y\colon E \to \{0,1\}$ of a multicut $y^{-1}(1)$ can be used to encode the decomposition induced by the multicut in an $|E|$-dimensional 01-vector
- For any $e \in E$, $y_e = 1$ indicates that an edge is cut, straddling distinct components



Multicut of a graph $G = (V, E)$

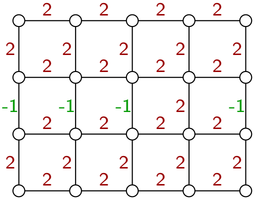
- The set of the characteristic functions of all multicuts of G:

$$Y_G := \left\{ y : E \to \{0,1\} \;\middle|\; \forall C \in \mathsf{cycles}(G)\, \forall e \in C :\; y_e \leq \sum_{f \in C \setminus \{e\}} y_f \right\}$$



Graph $G = (V, E)$

- An instance of the image decomposition problem is given by a graph $G = (V, E)$ and, for every edge $e = \{v, w\} \in E$, a (positive or negative) cost $c_e \in \mathbb{R}$ that is payed iff the incident pixels v and w are put in distinct components
- Such costs can be estimated, as we have seen, by means of logistic regression



Graph $G = (V, E)$. Edge costs $c : E \to \mathbb{R}$

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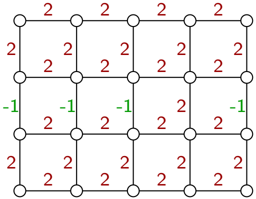
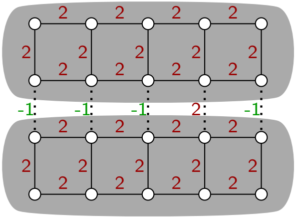


Graph $G = (V, E)$. Edge costs $c : E \to \mathbb{R}$

- Image decomposition problem:

$$\min_{y \in Y_G} \sum_{e \in E} c_e \, y_e$$

- The optimal solution is shown in the next slide



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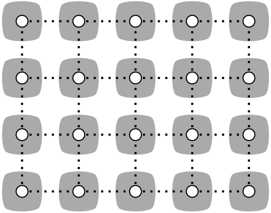
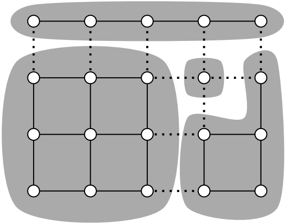
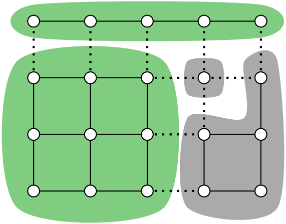


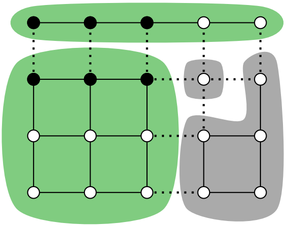
- One technique for finding feasible solutions to an image decomposition problem is **local search**.
- Starting from the finest decomposition into singleton components (depicted above), we greedily join neighboring components as long as this improves the cost (see next slide).



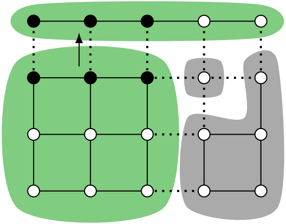
- Once no joining of neighboring components further reduces the cost, we consider all pairs of neighboring components



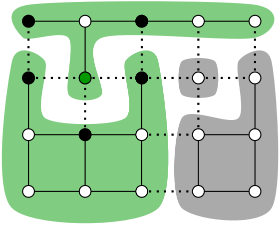
- Once no joining of neighboring components further reduces the cost, we consider all pairs of neighboring components (depicted in green)



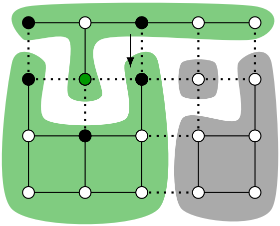
- Once no joining of neighboring components further reduces the cost, we consider all pairs of neighboring components (depicted in green) and all nodes at the shared boundary (depicted in black)



- Once no joining of neighboring components further reduces the cost, we consider all pairs of neighboring components (depicted in green) and all nodes at the shared boundary (depicted in black) and all possibilities of moving nodes from one component to the other.



- Once no joining of neighboring components further reduces the cost, we consider all pairs of neighboring components (depicted in green) and all nodes at the shared boundary (depicted in black) and all possibilities of moving nodes from one component to the other.
- The procedure is iterated until no such transformation further reduces the cost



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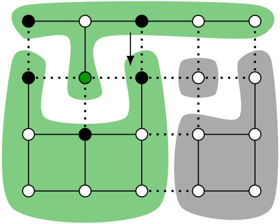

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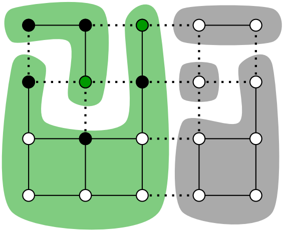

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Summary.

- We have seen an application of the correlation clustering problem to the task of image decomposition.
- This application is useful in settings where components of the image are indistinguishable by appearance, and where no prior knowledge can or shall be introduced on the number of size of components.