Computer Vision I

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Machine Learning for Computer Vision
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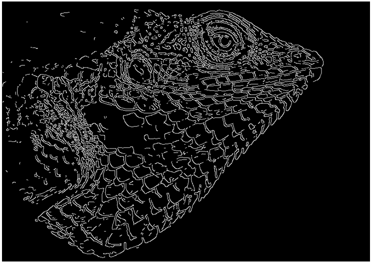
Edge and corner detection

Image[1]

Edge detection[1]



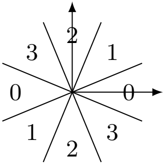
[1] https://en.wikipedia.org/wiki/Canny_edge_detector

Canny's edge detection algorithm[1] has four steps

1. Gradient computation from digital image $f \colon V \to \mathbb{R}$:

$$g = \sqrt{\partial_0 f + \partial_1 f} \qquad \texttt{std::hypot in C++} \tag{1}$$
$$\alpha = \operatorname{atan2}(\partial_1 f, \partial_0 f) \qquad \texttt{std::atan2 in C++} \tag{2}$$

2. Directional non-maximum suppression of g:



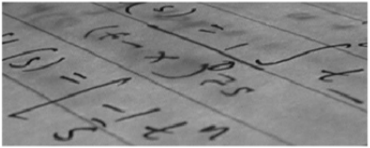
3	2	1
0		0
1	2	3

3. Double thresholding with $\theta_0, \theta_1 \in \mathbb{R}_0^+$ such that $\theta_0 \leq \theta_1$: A (any) pixel $v \in V$ is taken considered to be a **strong edge pixel** iff $\theta_1 \leq g(v)$ and is taken to be a **weak edge pixel** iff $\theta_0 \leq g(v) < \theta_1$.
4. Weak edge classification: A (any) pixel $v \in V$ is taken to be an **edge pixel** iff (i) v is a strong edge pixel, or (ii) v is a weak edge pixel and there is a strong edge pixel in the 8-neighborhood of v.

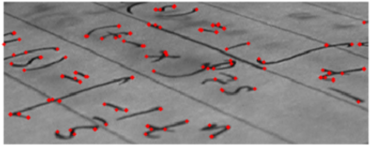
[1] J. Canny. A Computational Approach To Edge Detection. IEEE Transactions on Pattern Analysis and Machine Intelligence, 8(6):679–698, 1986

Edge and corner detection

Image[1]



Corner detection[1]



[1] https://en.wikipedia.org/wiki/Corner_detection

Definition 1. Let $n_0, n_1 \in \mathbb{N}$, let $V = [n_0] \times [n_1]$, let $f\colon V \to \mathbb{R}$ a digital image, let ∂_0, ∂_1 be discrete derivative operators, and let $N\colon V \to \mathbb{R}^V$.

For each $v \in V$:

- Let $A(v)$ be the $|N(v)| \times 2$-matrix such that for every $w \in N(v)$, we have

$$A_{w\cdot}(v) = ((\partial_0 f)(w), (\partial_1 f)(w)) \ . \tag{3}$$

- Let $k_v\colon N(v) \to \mathbb{R}_0^+$ such that $\sum_{w \in N(v)} k_v(w) = 1$.
- Define the **structure tensor** of f at v wrt. k_v as the 2×2-matrix

$$S_k(f)(v) := \sum_{w \in N(v)} k_v(w) A_{w\cdot}^T(v) A_{w\cdot}(v) \tag{4}$$

$$= \sum_{w \in N(v)} k_v(w) \begin{pmatrix} (\partial_0 f)^2(w) & (\partial_0 f)(w)(\partial_1 f)(w) \\ (\partial_0 f)(w)(\partial_1 f)(w) & (\partial_1 f)^2(w) \end{pmatrix} \ . \tag{5}$$

Remark 1. Fix a direction by choosing $r \in \mathbb{R}^2$ with $|r| = 1$ and consider the k_v-weighted squared projection of the gradient of the digital image:

$$\begin{aligned} P_r(v) &= \sum_{w \in N(v)} k_v(w) |A_{w\cdot}(v)\, r|^2 && (6) \\ &= \sum_{w \in N(v)} k_v(w)\, r^T A_{w\cdot}^T(v) A_{w\cdot}(v)\, r && (7) \\ &= r^T \left(\sum_{w \in N(v)} k_v(w) A_{w\cdot}^T(v) A_{w\cdot}(v) \right) r && (8) \\ &= r^T S(v)\, r && (9) \end{aligned}$$

With the spectral decomposition

$$S(v) = \sigma_1(v) s_1(v) s_1^T(v) + \sigma_2(v) s_2(v) s_2^T(v) \qquad (10)$$

we obtain

$$\begin{aligned} P_r(v) &= r^T \Big(\sigma_1(v) s_1(v) s_1^T(v) + \sigma_2(v) s_2(v) s_2^T(v) \Big)\, r && (11) \\ &= \sigma_1(v) |s_1(v) \cdot r|^2 + \sigma_2(v) |s_2(v) \cdot r|^2 \ . && (12) \end{aligned}$$

Remark 2.

- If $\sigma_1 = \sigma_2 = 0$, we have $P_r(v) = 0$ for any direction r. I.e. the image is constant.
- If $\sigma_1 > 0$ and $\sigma_2 = 0$, we can choose a direction r such that $P_r(v) = 0$. I.e. the gradient of the image is non-zero and constant.
- If $\sigma_1, \sigma_2 > 0$, we cannot choose r such that $P_r(v) = 0$. I.e. the gradient of the image varies across $N(v)$.

Definition 2. Let V the set of pixels of a digital image, let $S\colon V \to \mathbb{R}^{2\times 2}$ such that for any $v \in V$, $S(v)$ is the structure tensor of the image at pixel v, and let $\sigma_1(v) \geq \sigma_2(v) \geq 0$ be the eigenvalues of $S(v)$. **Harris' corner detector**[2] wrt. a neighborhood function $N\colon V \to 2^V$ refers to the function $\varphi_{\mathrm{NMS}} \circ \sigma_2$.

[2] C. Harris and M. Stephens. A Combined Corner and Edge Detector. Alvey Vision Conference. Vol. 15. 1988